Fin 410   **HW #6**
Name: Due Monday Sept 28th at midnight (upload to Learning Suite)
410 Section: 8am, 11am, or 3:30pm

Purpose for HW 6: **This homework is intended to build your understanding of the efficient frontier and the calculations we've been talking about in lecture. This homework takes time but in past years many students have indicated that they learned a lot from this particular assignment. For HW assignments you are welcome to talk to other students, get help from the TAs, or ask the professor. Submit your completed excel file on Learning Suite**

1) In this problem you will find the efficient investment weights for your specific virtual stock exchange portfolio. To do this, the directions below will walk you through the following big-picture steps: (A-B) downloading historical price information for the stocks in your portfolio, (C) formatting the downloaded price information, (D) calculating annual returns from the price data, (E) calculating the summary statistics for your specific assets that are necessary inputs to the frontier-generating formulas, (F) documenting expected returns and weights for your current portfolio, (G) creating 5000 hypothetical sets of weights to apply to the assets in your portfolio, (H and I) calculating the return and risk measures for 5000 hypothetical portfolios based on your assets and the hypothetical weights, (J-L) graphing the approximate frontier based on your specific assets, and (M) identifying the weights and dollars needed to make your virtual stock exchange portfolio more efficient. This assignment is excel intensive and will probably take you 2-5 hours to do. If you find yourself stumped on a step I recommend you go see the TAs (or talk to me) and ask how to do it rather than spend excessive amounts of time trying to figure it out and being frustrated. Don't wait until the last minute to try to do this homework. **Excel questions are much easier to answer in person rather than via email.** I've placed an example excel file on Learning Suite for my portfolio from a previous year. **You should format your spreadsheet for your portfolio in the same way as my file with the same tabs and sections so that when we grade them we know where to find the information**. Note that my example file is for 4 assets and you will need to do the same analysis with 5 assets. To hand in this assignment you will post your spreadsheet on Learning Suite. Please name your spreadsheet using your name so that when we download them the actual file name indicates whose file it is.

   A) **Get tickers:** Go to your virtual stock exchange account and write down the tickers for the 5 assets in your portfolio. Only use 5 assets for this assignment. For the assets you write down also write down the number of dollars you currently have invested in each. Calculate the proportion of your wealth associated with each asset. Assume that the total dollars you have invested in the 5 assets represents 100% of your wealth for this project. (i.e., ignore cash holdings.)

   B) **Get price information:** Go to http://finance.yahoo.com/ and enter the first ticker in the Quote Lookup field. Click the Go button. In the left column select Historical Prices. After the historical prices table appears select Monthly data and enter a Date Range that starts Jan 1, 2005 and ends Aug 31, 2015. Click Get Prices. (You are also welcome to download the data using applications you may have learned in other classes). See footnote 1 regarding tickers with price histories that don't go back to 1990.[1] Scroll to the bottom of

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[1] For this project, you will create an excel file with a separate column of monthly data for each of your tickers. These columns will be placed side-by-side. For the tickers with price histories that go back only a couple of years

1

the page and click Download to Spreadsheet. In the downloaded spreadsheet delete all columns except for Date and Adj Close. Rename the column title "Adj Close" to the ticker symbol. You will repeat this step for each of the tickers written down in part A.

C) **Place information in one spreadsheet (see the "data" tab in the example spreadsheet):** After you have downloaded the adj closing prices for each of the tickers, copy all of them to a single worksheet. Eliminate the redundant date columns so that you have only one date column and 5 columns titled by their tickers (see example spreadsheet). Be sure when you copy the price data that the prices for the various tickers on the same date end up on the same row. I've posted my version of the spreadsheet (hw6_prof_spreadsheet_4_ assets_2012.xlsm) on Learning Suite as an example. Refer to the Data tab for this step. You will notice in my file if you scroll down that 3 of my securities only have price information for a few years.

D) **Calculate monthly and annualized returns:** After you have the adj closing prices for your tickers side-by-side on one spreadsheet, create month-to-month return columns next to them (see my example spreadsheet). The return formula is $(P_1-P_0)/P_0$ at the monthly level. Once you have created the return formula for one month for one ticker you can copy the formula down and across to calculate the same formula for the other dates and tickers. Be aware of whether the earlier or later dates are at the top of the spreadsheet because it will affect your return formulas. After you have the monthly returns create columns of annualized returns next to them (see my example spreadsheet). Annualize the returns as APRs for simplicity.

E) **Calculate summary information that will be used as inputs for the efficient frontier:** Calculate the average annual return and standard deviation for each of your tickers. Place these in a table as shown in my spreadsheet. Create a correlation matrix as shown in the example spreadsheet. To do this you can click on Data/Data Analysis in Excel. Select Correlation from the drop-down menu. Click Okay. For the input range select all the annual returns including the title columns (on my example spreadsheet the input range was L2:O255 but with 5 assets your range will be different.) Indicate on the menu that labels are included on the first row. Click on the Output Range circle and then indicate the cell where you want the correlation matrix to be placed.

F) **Write information from part (A) in spreadsheet:** Write the information from (A) below the correlation table as shown in my example spreadsheet (tickers, dollars in each ticker, overall number of dollars, etc.). Calculate the expected return for your current portfolio (assuming historical averages predict future returns). Clearly label the weights. We are working with the assumption that historical average annual returns on our assets represent the expected annual returns going forward.

G) **Create 5000 hypothetical weights:** To do this step create a new tab in the spreadsheet and call it "weights and frontier" (see example spreadsheet). You will create in this worksheet 5000 sets of hypothetical weights for your assets (with each set on a new row). In my example spreadsheet I have 4 assets in my portfolio so I have 4 columns of weights. Label the weight columns using your tickers. Before creating the hypothetical weights, in the first row of the weights table enter your current portfolio weights that you solved for in (A). Below these actual weights you will create 5000 rows of hypothetical weights. To create the hypothetical weights, in the second row of the weights table in the column for

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some of the rows will remain blank. Be sure that the price data for each of the stocks is for the same month in a given row of data. If you look at my file you will see that some of my stocks only go back a few years.

the first asset type the formula "=(RAND())*1.5 – 0.3". (The first row in the set of weights has your actual weights so the second row will have the first set of hypothetical weights). This formula first picks a random number from a uniform distribution between 0 and 1. This number is then multiplied by 1.5. We do this so that some weight positions can be larger than 1. We then subtract .3 from it to allow for some shorting. This approach will generate random weights generally between -.2 and 1.2. You will copy this formula ("=(RAND())*1.5 – 0.3") across the second row for each of the cells in the weights table until you get to the last asset's column. <u>In the last column of weights the formula will be 1 minus the sum of the other weights</u>. This makes the weights sum to 1 across the row. See my example spreadsheet if you need to see the formulas but remember that my spreadsheet uses 4 assets and yours should use 5. After you have the correct formulas for the weights across the second row you will then copy them and paste them over the 4999 rows below the second row. Note that each of the 5000 rows below the top row will then contain a set of hypothetical weights for your assets. We will use these 5001 sets of weights to calculate the expected return and standard deviation combinations available to you based solely on the assets currently in your portfolio based on different investment weights. After you are done creating the 5001 rows of weights you will copy them in the rows to the right **as values** so that the formulas don't keep updating your weights each time you make changes to the spreadsheet. To paste as values do the following: highlight the 5001 rows and the 5 columns that have the weights, press ctrl-c to copy, right click on an empty cell to the right of these columns and select paste special. Then choose to paste as values. You can see this in my spreadsheet where columns B-E update (except for the actual weight ros) each time the spreadsheet is opened but columns G-J do not).

H) **Calculate expected return for each of the hypothetical portfolios:** To the right of the columns of hypothetical weights calculate the expected portfolio return for those weights. Use the weights that were pasted as values to calculate the expected returns. The general formula for 5 assets would be $E[R_p] = w_1E[R_1] + w_2E[R_2] + w_3E[R_3] + w_4E[R_4] + w_5E[R_5]$.

I) **Calculate the standard deviation for the hypothetical portfolios:** To the right of the expected returns calculate the standard deviation for each hypothetical portfolio. For the weights be sure to use the weights that were pasted as values. <u>You only need to type in the (variance) formula **once** at the top of the column and then copy it down for the 5000 hypothetical portfolios</u>. Be sure when you type the formula in excel that you use $ references for the standard deviations and correlations so that when you copy the formula only the cell references to the weights update. (Talk to the TAs or to me if this does not make sense.) The formulas for the variance of a portfolio of 3, 4, and 5 risky assets are shown below. You will use the formula that applies to your portfolio with 5 assets. You will need to type this formula (**once**) at the top of the column using references to the actual standard deviations and expected returns you calculated for your assets in part (E). The $ references will appear in front of the standard deviations and the correlations so that when you copy and paste the formula down the column only the weights update for each row. (As a check on your formula ensure that none of the calculated variances are negative).

i. **3 assets (A,B,C):** $\sigma_{Rp}^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + 2w_Aw_B\sigma_A\sigma_B\rho_{AB} + 2w_Aw_C\sigma_A\sigma_C\rho_{AC} + 2w_Bw_C\sigma_B\sigma_C\rho_{BC}$

ii. **4 assets (A,B,C,D):** $\sigma_{Rp}^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + w_D^2\sigma_D^2 + 2w_Aw_B\sigma_A\sigma_B\rho_{AB} + 2w_Aw_C\sigma_A\sigma_C\rho_{AC} + 2w_Aw_D\sigma_A\sigma_D\rho_{AD} + 2w_Bw_C\sigma_B\sigma_C\rho_{BC} + 2w_Bw_D\sigma_B\sigma_D\rho_{BD} + 2w_Cw_D\sigma_C\sigma_D\rho_{CD}$

iii. **5 assets(A,B,C,D,E):** $\sigma^2_{Rp} = w^2_A\sigma^2_A + w^2_B\sigma^2_B + w^2_C\sigma^2_C + w^2_D\sigma^2_D + w^2_E\sigma^2_E + 2w_Aw_B\sigma_A\sigma_B\rho_{AB} + 2w_Aw_C\sigma_A\sigma_C\rho_{AC} + 2w_Aw_D\sigma_A\sigma_D\rho_{AD} + 2w_Aw_E\sigma_A\sigma_E\rho_{AE} + 2w_Bw_C\sigma_B\sigma_C\rho_{BC} + 2w_Bw_D\sigma_B\sigma_D\rho_{BD} + 2w_Bw_E\sigma_B\sigma_E\rho_{BE} + 2w_Cw_D\sigma_C\sigma_D\rho_{CD} + 2w_Cw_E\sigma_C\sigma_E\rho_{CE} + 2w_Dw_E\sigma_D\sigma_E\rho_{DE}$

J) **Find the maximum return for ranges of standard deviation values:** In your excel file you will identify the maximum expected return for several small ranges of standard deviation values. The big-picture goal here is to be able to graph the expected return and standard deviations of all 5001 portfolios. Remember that in the graph the x-axis is standard deviation. So essentially you are asking yourself what the maximum expected return portfolio (i.e. a frontier point) is in your data as you move over small intervals of the x-axis. See my example spreadsheet on the "weights and frontier" tab.[2] This step allows us to (roughly) trace out the efficient frontier based on the 5000 portfolios we created. So for this step, find the <u>maximum</u> expected return for each of the following ranges of standard deviation: 0 to .1, .1 to .2, .2 to .4, .4 to .6, .6 to .9, .9 to 1.2, 1.2 to 1.5, 1.5 to 2, 2 to 2.5, 2.5 to 3, and 3 to 3.5. Also find the <u>minimum</u> expected return for each of the following ranges of standard deviation: .9 to 1.2, .6 to .9, .4 to .6, .2 to .4. To find these maximums and minimums you will use formulas <u>shown and explained in the example spreadsheet</u>. In

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[2] Last year several of the students' excel files had trouble with the conditional Min/Max formulas used in HW 6 as shown in column W of the "weights and frontier" spreadsheet in the example HW 6 file. I have a couple of comments about these formulas.

First, assume that your E[r$_p$]'s are listed in column L so a reference to $L3:$L5004 is a reference to your expected returns. Assume also that your standard deviations are listed in column N so a reference to $N3:$N5004 is a reference to your standard deviations. Also assume that you are interested in calculating the min or max expected returns over a specific range of standard deviation values (i.e., .4 and .6).

The original syntax for these formulas as shown in my spreadsheet is as follows:
{=MAX(($L$3:$L$5004)*($N$3:$N$5004>=.4)*($N$3:$N$5004<.6))}

This expression is meant to look for the max expected return value (in L column) that corresponded with a standard deviation (in N column) that was larger than .4 and smaller than .6. The min function worked the same way but using Min instead of Max. This array-based formula requires that you click at the right-most side of the formula in the formula row after typing the formula and then pressing ctrl-shift-enter. After doing this the curly brackets appear automatically around the formula (don't manually type the curly brackets.)

Several of the students' spreadsheets wouldn't calculate this value for some reason. Most of the problems occurred with the Min formulas and not the Max formulas. I have two suggestions for you if your conditional Min (or Max) formulas are not working:
(1) double check your variance formula to make sure you have no errors in your variance formula.
(2) try a different syntax for the conditional min/max formulas. I suggest you try the following alternative syntax if the original is not working for you:

=MIN(IF(($N$3:$N$5004>=.4)*($N$3:$N$5004<.6),$L$3:$L$5004))

In this new syntax you are telling excel to take the min (or max) of the expected returns (in column L) if the standard deviations (in column N) are larger than .4 and smaller than .6. Using the nested IF condition in the Min function seemed to work on the spreadsheets that I saw that had problems in the Min formulas using the original syntax. For this formula you also have to click at the right-most side of the formula in the formula bar and press ctrl-shift-enter for it to work (and for it to add the curly brackets shown in my spreadsheet).

4

my example spreadsheet if you look at the plot you will see that no points appear above the 0 to .1 standard deviation range so I do not use that range but your portfolio may be slightly different.

K) **Write the x and y coordinates that will be used to graph the efficient frontier line in a separate table:** After you have solved for the Min/Max returns in part (J) for the ranges of standard deviation you need to type or copy/paste the Min/Max values (y-coordinates are Expected returns) along with a (single) standard deviation value for each range that will be used as the x-coordinate into a table. See my example file. You can pick any standard deviation point within the range to be the representative x-coordinate. Be sure not to include Min/Max positions of "0" for ranges of standard deviation where none of your portfolio returns occurred. In my example I didn't include 3 points because of this (see cells with 0.0000 written in red in column w. They don't appear in the part k table).

L) **In one graph plot the E[r] and σ values for the 5001 portfolios from (H) and (I), plot the frontier line based on the points from (K), and plot the position of your current portfolio.** To do this step you will need to plot 3 separate "data series" in excel. When you graph the 5001 portfolios you will need to decrease the size of the markers so that you can easily see each portfolio. When you add the frontier line you will need to change the chart type from scatter to line. Please increase the marker size for your current portfolio and clearly label it in excel. Is your current portfolio on the efficient frontier given your assets? (Note that some students have more experience with excel than others. This step should not take you more than 10 minutes. If it is taking more time than that then please come to my office so that I can help you.)

M) **Assume that (1) you want the same assets in your portfolio as you currently have and that (2) you want the same risk as your current portfolio has but that now you want to pick the dominant portfolio given these requirements.** Assume for this problem that you can't borrow or lend at the risk-free rate. Add a circle to the graph to show where the dominant portfolio would appear with the same risk as your current portfolio. Find the weights of one of the 5001 portfolios in this circle or close to it and type the weights below the chart (if you place your cursor over a point on the graph it should give you the standard deviation and E[r] of the point so you can then search for these values in the return and deviation columns using ctrl-f. When using the ctrl-f you may need to select Look In = values in the search options to find the numbers). Note that in this step I am not asking you to use math to find the dominant portfolio weights. Rather I am asking you to identify one of the 5001 portfolios that is close to where the dominant one would appear on your frontier. Below the dominant weights in the file calculate how many dollars you should have in each asset.

N) **Draw the capital allocation line (i.e. manually insert a shape –line—in excel) that is tangent to the efficient frontier you created.** In this step you are assuming that you can now borrow and lend. Assume for the CAL that the risk free rate is 3%. Label the tangency portfolio. Is there a portfolio that could exist on the CAL that has the same risk as your current portfolio and higher expected return than both your current portfolio and the portfolio you highlighted in part M? Draw a triangle for this portfolio.

O) In words answer these questions in your spreadsheet under the plot:
    1. **What asset is shorted in order to create a portfolio on the CAL tangent to the efficient frontier for a portfolio located above and to the right of the tangency portfolio?**

2. **Is the risk free asset needed in order to invest in the tangency portfolio?**
3. **Is shorting required to create an asset on the CAL tangent to the efficient frontier for a portfolio located below and to the left of the tangency portfolio?**
4. **Is the tangency portfolio the same as the minimum variance portfolio?**
5. **Explain what we accomplished by creating 5000 random weights.**
6. **Which 2 equations are used to plot all the points on the plot?**
7. **If we wanted to not allow shorting of any kind what would we change about this assignment?**

$$(1+y)/y \qquad\qquad \frac{1+y}{y} - \frac{n}{(1+y)^n - 1} \qquad\qquad \frac{1+y}{y} - \frac{(1+y) + n(c-y)}{c[(1+y)^n - 1] + y}$$

$$PV = \frac{FV}{\left(1 + \frac{r}{m}\right)^{m*n}} \qquad\qquad FV = PV\left(1 + \frac{r}{m}\right)^{m*n} \qquad\qquad PV = \frac{CF}{r}$$

$$PV = \frac{CF_1}{r - g} \qquad\qquad PV = CF\left[\frac{1}{\frac{r}{m}} - \frac{1}{\frac{r}{m}\left(1 + \frac{r}{m}\right)^{m*n}}\right]$$

$$E[r_j] = r_f + \beta_j\big(E[r_m] - r_f\big) \qquad\qquad r = \frac{r_1 + r_2 + \cdots + r_n}{n}$$

$$D^* = \frac{D}{1 + y} \qquad\qquad \frac{\Delta Bond\ Price}{Bond\ Price} = -D^*(\Delta y) = -\frac{D}{1 + y}(\Delta y)$$

$$\text{Value} = \sum_{t=1}^{t=\infty} \frac{CF_t}{(1+r)^t} \qquad\qquad \frac{\Delta Bond\ Price}{Bond\ Price} = -D^*(\Delta y) + (.5)(\text{Convexity})(\Delta y)^2$$

$$Convexity = \frac{1}{Price(1+y)^2}\sum_{t=1}^{n}\frac{CF_t}{(1+y)^t}\left(t^2 + t\right)$$

$$\frac{E[r_p] - r_f}{\sigma_p} \qquad\qquad \sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\sigma_x\sigma_y\rho_{xy}$$

$$r = [(1 + r_1)(1 + r_2) \dots (1 + r_n)]^{1/n} - 1$$

$$E[r] = \sum_{i=1}^{n} p(r_i)r_i \qquad\qquad (1+ \text{real rate})\ (1+ \text{inflation rate}) = (1+ \text{nominal rate})$$

$$Var(r) = \sum_{s=1}^{S} p(s)(r(s) - E[r])^2 = \sigma^2 \qquad\qquad \rho_{sb} = \frac{Cov(r_s, r_b)}{\sigma_s\sigma_b}$$

$$E[z] = aE[x] + bE[y] \qquad\qquad D = \sum_{t=1}^{n} t*(w_t) = \sum_{t=1}^{n} t*\left(\frac{\frac{CF_t}{(1+y)^t}}{Bond\ Price}\right)$$

$$\text{ave}\left[\frac{(r-\bar{r})^3}{\sigma^3}\right] \qquad\qquad \text{ave}\left[\frac{(r-\bar{r})^4}{\sigma^4}\right] - 3$$