<u>Time Value of Money Review Notes:</u>

This worksheet provides a review of the time value of money formulas you might have learned in other classes. All of the formulas discussed in this note were also discussed during the lecture. If you already know how to use these formulas from the lecture then you probably won't need to spend much time with this review. On the other hand, if you had trouble understanding the time value of money lecture then please spend time thinking about the examples discussed below.

By the end of this review, you should be familiar with the 4 equations that do the following 4 tasks:

1. Discount a future value to a present value. This formula is useful in moving a single cash flow back in time.
2. Convert a present value to a future value. This formula is useful in moving a single cash flow forward in time.
3. Calculate the present value or payment associated with an annuity. This formula is useful in valuing a finite stream of regular cash flows.
4. Calculate the present value or payment associated with a perpetuity. This formula is useful in valuing an infinite stream of regular cash flows.

(1) To discount a future value (FV) to a present value (PV) use the following formula:

$$\mathrm{PV} = \frac{\mathrm{FV}}{(1+\mathrm{r})^{\mathrm{n}}} = FV\ multiplied\ by\ the\ "n\ year\ discount\ factor"$$

where r is the annual discount rate and n is the number of years. In this class we will assume that the discount rates are always compounded annually. If you are ever in a setting where the interest is being compounded (or discounted) at a higher frequency each year (e.g., credit cards tend to assess interest monthly which is "monthly compounding") then you need to adjust all of the formulas used in this review sheet.

Example: Would you prefer \$100 today or \$110 next year if the appropriate discount rate is 6%? One way to answer this question would be to calculate the present value of the \$110 and then compare that PV with the \$100 (which is already in today's dollars). The present value of the \$110 would be

$$\mathrm{PV} = \frac{\mathrm{FV}}{(1+\mathrm{r})^{\mathrm{n}}} = \frac{110}{(1+.06)^{1}} = \$103.77$$

In this problem the \$110 are in year 1 and need to be multiplied by a "1-year discount factor" to convert them to year 0 dollars.

(2) To convert a present value to a future value, use the following formula:

$$\mathrm{FV} = \mathrm{PV}(1+r)^{n}$$

where r is the annual interest rate and n is the number of years.

Example: How much would \$100 be worth in 50 years with 6% annual interest? The answer to this question is the future value of \$100 after it experiences 50 years of interest. The formula would be $FV = PV(1+r)^n = 100(1.06)^{50} = \$1{,}842.015$.

In this example the \$100 is in year 0 and is multiplied by a "50-year interest factor" to convert it to year 50 dollars.

(3) To calculate the present value of an annuity that pays a fixed payment (PMT) over time use the following formula:

$$PV\ of\ annuity = PMT\left[\frac{1}{r} - \frac{1}{r(1+r)^n}\right]$$

where r is the annual interest rate and n is the number of years over which you receive an annual payment of PMT. In the lecture slides "CF" is used instead of "PMT" to represent reoccurring cash flows.

Example: (Note -This example assumes you understand the cash flows associated with coupon bonds. We will review coupon bonds in Lecture #3. So if you don't know about the cash flows tied to coupon bonds then skip this example until after the next lecture). Assume you own a 10% coupon bond that has a par value of \$1000 and pays an annual coupon. The bond has 10-years until maturity. Assume the discount rate is 6%. What is the present value of the stream of coupon payments (i.e., not including the final \$1000 payment)? What is the price of the bond?

To answer the first question use the formula

$$PV = PMT\left[\frac{1}{r} - \frac{1}{r(1+r)^n}\right] = 100\left[\frac{1}{.06} - \frac{1}{.06(1+.06)^{10}}\right] = \$736.008$$

So the present value of a 10-year annuity of coupon payments in this example is worth \$736.01. To find the price of the bond we need to then add the present value of the final \$1,000 payment that will occur 10 years in the future to the present value of the 10 coupon payments. To calculate the present value of the single cash flow at the end of the 10 years we use the formula

$$PV = \frac{FV}{(1+r)^n} = \frac{1000}{(1+.06)^{10}} = \$558.395$$

So the price of the bond equals the present value of the coupon payments plus the present value of the final \$1000 payment (bond price = \$736.01 + \$558.395).

Example: Congratulations! You just won the lottery… but you have to choose between 2 possible payment options (for simplicity assume all values are after-tax):

1) Get a lump sum of \$38 million today, or
2) Get paid in 30 annual installments of \$3 million each year, <u>starting today</u>

For this problem assume your discount rate is 7%. Which option should you choose?

Here’s a timeline of the cash flows:

	Year 0 (today)	1	2	3	…	28	29	30
Option 1	38							
Option 2	3	3	3	3	…	3	3	0

We know that the PV of the first option is $38 million because the $38 million is already in year 0 dollars. We need to calculate the PV of the other option in order to compare the 2 options.

The 2nd option is a 30-year “annuity due” which means the first payment occurs today as opposed to one year in the future. Another way to think about the 2nd option is that it is a standard 29-year annuity with an additional payment added on today. In a standard annual annuity the first payment is 1 year in the future. In an “annuity due” the first payment occurs today.

So for the 2nd option we need to discount all the 29 future years’ dollars to year 0 dollars so we can then compare the present value of all the 2nd option’s payments with the $38 million present value of the 1st option. The first $3 million payment is already in year 0 dollars so we only need to move the other 29 payments back to year 0.

PV of 30 payments = the first $3 million payment plus the PV of the other 29 payments.

$$PV = 3 + 3\left(\frac{1}{0.07} - \frac{1}{0.07(1.07)^{29}}\right) = \$39.833$$

So in comparing the 3 options the comparison is between 38M and 39.83M. The second is the best option assuming that on time value of money concerns apply.

(4) To calculate the present value of a perpetuity that pays a fixed payment forever use the following formula:

$$PV\ of\ perpetuity = \frac{PMT}{r} = \frac{CF}{r}$$

Example: Congratulations! You have been successful in business and now want to give back to your former university by creating a scholarship. Assume that your gift will grow at 8% a year forever. How much money would you need to give to the university today in order to provide a $20,000 annual scholarship each year forever? In this example, the r = 8% and the $20,000 is the reoccurring cash flow.

$$PV\ of\ perpetuity = \frac{PMT}{r} = \frac{\$20{,}000}{.08} = \$250{,}000$$

So you would need to give $250,000 today to create a perpetuity of $20,000 payments assuming an annual rate of 8%.